Axial anomaly and the δ_{LT} puzzle

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The axial anomaly contribution to generalized longitudinal-transverse polarizability δ_{LT} is calculated within Regge approach. It is shown that the contribution from the exchange of the $a_1(1260)$ Regge trajectory is nontrivial and might have the key role to explain the large difference between the predictions of chiral perturbation theory and the experimental data for the neutron δ_{LT}. We also present the prediction for the proton δ_{LT} that will be measured at the Thomas Jefferson National Accelerator Facility in near future.

The measurement of spin-dependent lepton-nucleon cross sections provides very important information about nucleon structure and is a very useful tool for examining the validity of various approaches for description of the QCD effects at large distances between quarks and gluons. One of such successful approaches is chiral perturbation theory (χPT) that enjoys successful description of hadron physics at small momentum transfer and at low energy. However, it was found that the recent predictions [1, 2] of various versions of χPT show a strong deviation from the data for generalized longitudinal-transverse polarizability (δ_{LT}) of the neutron at low Q^2 measured by the E94010 Collaboration of the Thomas Jefferson National Accelerator Facility (TJNAF) [3]. The generalized longitudinal-transverse polarizability can be evaluated from a combination of the spin-dependent structure functions g_1 and g_2, and is an ideal quantity to test models for hadron reactions at low Q^2. Therefore, such a large deviation is a serious challenge to the χPT approach to low energy reactions, and finding the possible sources for this discrepancy is an important open question.

In this paper, we suggest a possible way to resolve this puzzle based on the consideration of the axial anomaly contribution to δ_{LT} arising through the t-channel a_1 Regge-trajectory exchange to the spin-dependent forward scattering amplitude for $\gamma^* N \to \gamma^* N$. We will show that this contribution is nontrivial and has a crucial role to bring a χPT prediction for the neutron to the measured data. For further testing of this idea, we also present the prediction for the proton δ_{LT} in this approach, which can be examined by the planned experiment at TJNAF [4].

It is widely known that the axial anomaly [5, 6] plays a very important role in hadron physics. Originated from the quark triangle diagram, the axial anomaly determines the $\pi^0 \to \gamma\gamma$ decay width and provides a crucial role to the "spin crisis." (For a review, see, for example, Ref. [7].) In the present paper, we calculate its contribution to δ_{LT}, which is induced by the t-channel exchange of the $a_1(1260)$-meson Regge trajectory as depicted by the diagram in Fig. 1.

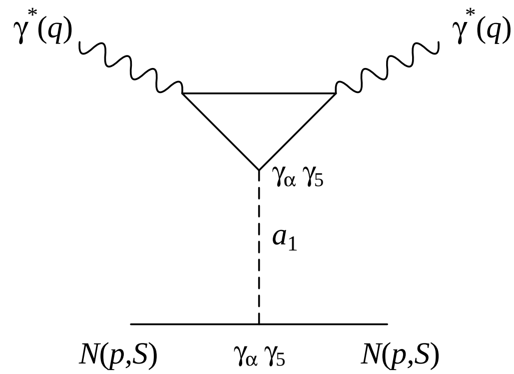

FIG. 1. The a_1-exchange contribution to the virtual photon-nucleon forward scattering amplitude.

In the hadronic tensor of two electromagnetic currents,

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4x\, e^{ipx} \langle p\, S \mid [J_\mu(x), J_\nu(0)] \mid p\, S\rangle, \quad (1)$$

the spin-dependent part is

$$W_{\mu\nu}^{\rm spin} = i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha S^\beta}{(p\cdot q)}\, g_1(x,Q^2) + i\epsilon_{\mu\nu\alpha\beta} \frac{q^\alpha \left[(p\cdot q)S^\beta - (S\cdot q)p^\beta\right]}{(p\cdot q)^2}\, g_2(x,Q^2), \quad (2)$$

where q and p are the momenta of the virtual photon and the nucleon, respectively. In Eq. (2), $x = Q^2/(2p\cdot q)$ with $Q^2 = -q^2$, and $g_1(x,Q^2)$ and $g_2(x,Q^2)$ are spin-dependent nucleon structure functions. The spin vector of the nucleon S_μ is normalized as $S^2 = -M_N^2$ with M_N being the nucleon mass. The details can be found, for example, in Ref. [8]. In terms of these structure functions the generalized longitudinal-transverse polarizability δ_{LT} can be written in the following form:

$$\delta_{LT}(Q^2) = \frac{16\alpha M_N^2}{Q^6} \int_0^{x_0} dx\, x^2 \left[g_1(x,Q^2) + g_2(x,Q^2)\right], \quad (3)$$

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where $x_0 = Q^2/(2M_N\nu_0)$, and $\nu_0 = m_\pi + (m_\pi^2 + Q^2)/(2M_N)$ is the threshold photon energy for π-meson production. Here, $\alpha = e^2/4\pi$ is the electromagnetic fine structure constant.

The spin-dependent part of the forward Compton amplitude is

$$T_{\mu\nu}^{\rm spin} = i\epsilon_{\mu\nu\alpha\beta}\frac{q^\alpha S^\beta}{M_N^2}A_1(Q^2,\nu) + i\epsilon_{\mu\nu\alpha\beta}\frac{q^\alpha\left[(p\cdot q)S^\beta - (S\cdot q)p^\beta\right]}{M_N^4}A_2(Q^2,\nu), \tag{4}$$

which is related to the structure functions g_1 and g_2 as

$$g_1(x,Q^2) = \frac{(p\cdot q)}{2\pi M_N^2}\,\mathrm{Im}\left[A_1(Q^2,\nu)\right],$$
$$g_2(x,Q^2) = \frac{(p\cdot q)^2}{2\pi M_N^4}\,\mathrm{Im}\left[A_2(Q^2,\nu)\right], \tag{5}$$

where ν is the photon energy. Since we are interested in the behavior of δ_{LT} at low Q^2, it is rather convenient to use the variable ν instead of the variable x defined for deep inelastic scattering. Then, the generalized longitudinal-transverse polarizability δ_{LT} of Eq. (3) can be written as

$$\delta_{LT}(Q^2) = \frac{2\alpha}{M_N}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu^4}\left[g_1(\nu,Q^2) + g_2(\nu,Q^2)\right]. \tag{6}$$

The contribution of the diagram in Fig. 1 to the spin-dependent forward Compton scattering amplitude is

$$T_{\mu\nu}^{\mathrm{spin},a_1} = g_{a_1NN}\bar{N}(p,S)\gamma_\beta\gamma_5 N(p,S) \times P_{a_1}^{\alpha\beta}(t=0,\nu)R_{\mu\nu\alpha}(Q^2), \tag{7}$$

where g_{a_1NN} denotes the coupling constant of the a_1-meson coupling to the nucleon, $P_{a_1}^{\alpha\beta}(t,\nu)$ is the a_1-meson propagator, and $R_{\mu\nu\alpha}(Q^2)$ is the triangle part of the diagram in Fig. 1, which is related to the axial anomaly. For the point-like a_1-quark vertex in the triangle graph[1] we use the well-known formula [7],

$$R^{\mu\nu\alpha}(Q^2) = i\frac{\mathcal{A}}{2\pi^2}g_{a_1qq}q_\tau\epsilon^{\mu\nu\tau\alpha}F(Q^2,m_q^2), \tag{8}$$

where $\mathcal{A} = N_c(e_u^2 - e_d^2)/\sqrt{2}$. Here, N_c is the number of color, e_q is the electric charge of q-quark, g_{a_1qq} is the coupling constant of the a_1 meson and the quark, and m_q is the constituent quark mass in the triangle diagram. In Eq. (8), the form factor $F(Q^2,m_q^2)$ is

$$F(Q^2,m_q^2) = 1 + \frac{2m_q^2}{Q^2\rho(Q^2)}\log\left(\frac{\rho(Q^2)-1}{\rho(Q^2)+1}\right), \tag{9}$$

where

$$\rho(Q^2) = \sqrt{1+\frac{4m_q^2}{Q^2}}. \tag{10}$$

Note that this form factor vanishes in the real photon limit, i.e., $F(Q^2,m_q^2)\to 0$ as $Q^2\to 0$. This feature is in agreement with the Landau-Yang theorem [10, 11] that prohibits the axial-vector meson decay into two real photons. Therefore, the a_1-meson exchange does not contribute to the real photon forward Compton scattering amplitude. As a result, the a_1 exchange does not alter, for example, the Gerasimov-Drell-Hearn sum rule and δ_{LT} at $Q^2 = 0$.

At high center-of-mass photon-nucleon energy, the a_1-meson propagator should be replaced by its Regge propagator as [12]

$$\frac{g^{\alpha\beta} - k^\alpha k^\beta/M_A^2}{t - M_A^2} \Rightarrow P_A^{\alpha\beta}(t,\nu)_{\rm Regge} \tag{11}$$

where A stands for the a_1 meson, and

$$P_A^{\alpha\beta}(t,\nu)_{\rm Regge} = \left(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{M_A^2}\right)\left(\frac{s}{s_0}\right)^{\alpha_A(t)-1} \times \frac{\pi\alpha'_A}{\sin[\pi\alpha_A(t)]}\frac{\sigma_A + e^{-i\pi\alpha_A(t)}}{2\Gamma[\alpha_A(t)]}. \tag{12}$$

Here, $s \approx 2p\cdot q = 2M_N\nu$, $t = (p-q)^2$, and $s_0 \approx 4$ GeV^2 [13, 14]. The signature of the a_1 trajectory is $\sigma_A = -1$ and the Regge trajectory of the a_1-meson is given by

$$\alpha_A(t) = \alpha_A(0) + \alpha'_A t. \tag{13}$$

We assume that the a_1-trajectory is an ordinary Regge trajectory with slope $\alpha'_A \approx 0.9$ GeV^{-2}. Then the intercept of the trajectory is estimated as $\alpha_A(0) \approx -0.36$ for the a_1-meson mass $M_A = 1.23$ GeV [15].

By making use of the relation,

$$\bar{N}(p,S)\gamma_\beta\gamma_5 N(p,S) = 2S_\beta, \tag{14}$$

the contribution of the a_1-trajectory exchange to $g_1 + g_2$ is obtained as

$$g_1(\nu,Q^2) + g_2(\nu,Q^2) = -g_{a_1qq}g_{a_1NN}\mathcal{A}\frac{M_N\nu\alpha'_A}{4\pi^2\Gamma[\alpha_A(t)]} \times \left(\frac{2M_N\nu}{s_0}\right)^{\alpha_A(0)-1}F(Q^2,m_q^2). \tag{15}$$

Substituting Eq. (15) into Eq. (6) and then performing the integration over ν leads to the final result for the a_1

[1] For large values of $Q^2 \geq 1$ GeV one should take into account the non-locality of this vertex, which will lead to the suppression of the form factor at large Q^2. There can be another form for the a_1-quark vertex as suggested in Ref. [9], which, however, does not contribute to the forward Compton scattering amplitude.

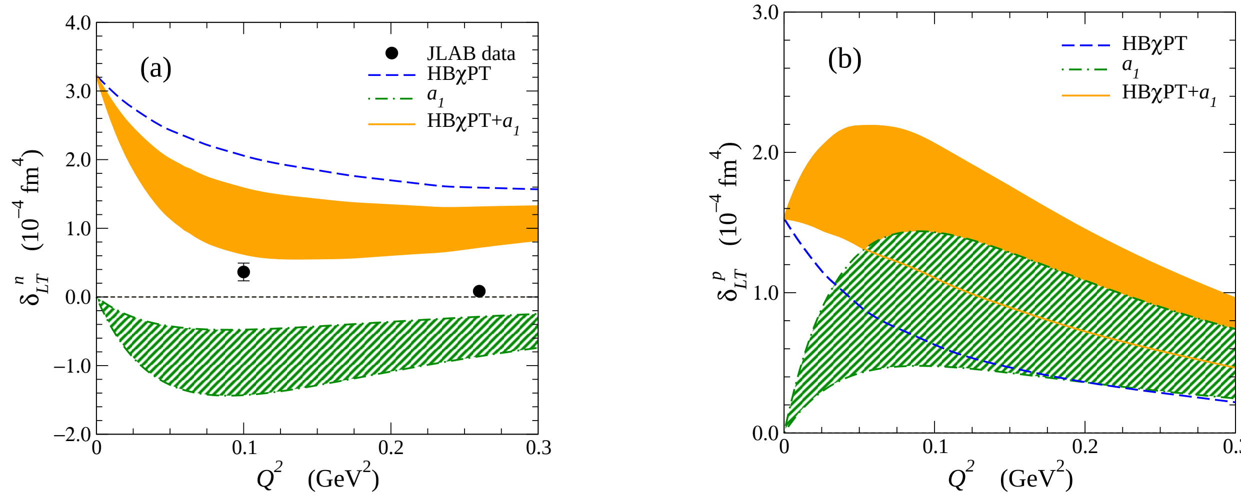

FIG. 2. (a) Contribution of the a_1-exchange to the generalized longitudinal-transverse polarizability of the neutron δ_{LT}^n as a function of Q^2 (shaded area). The dashed line is the result of heavy baryon χPT of Ref. [1]. The filled area is the sum of the prediction of Ref. [1] and the a_1-exchange contribution obtained in this work. The experimental data are from Ref. [3]. (b) Same for the generalized longitudinal-transverse polarizability of the proton δ_{LT}^p.

contribution to δ_{LT}, which reads

$$\delta_{LT}^{n,p} = \pm \frac{3\alpha g_{a_1pp}^2 \alpha_A' 2^{\alpha_A(0)-2.5}}{5\pi^2\Gamma[\alpha_A(0)]\{3-\alpha_A(0)\}M_N^2 z_0^{3-\alpha_A(0)}} \times \left(\frac{M_N^2}{s_0}\right)^{\alpha_A(0)-1} F(Q^2, m_q^2), \tag{16}$$

where $z_0 = \nu_0/M_N$, and we used the constituent quark model relation [16]

$$g_{a_1qq} = \frac{3g_{a_1pp}}{5}. \tag{17}$$

The upper and the lower sign in Eq. (16) correspond to the neutron and the proton case, respectively. In Fig. 2, our results for the a_1-exchange contribution to the generalized longitudinal-transverse polarizability δ_{LT} of the nucleon are presented with the coupling constant

$$g_{a_1pp} = 7.15 \pm 1.92 \tag{18}$$

that is obtained by using the assumption of axial-vector dominance [17], which gives the relation

$$\frac{g_A}{g_V} = \frac{\sqrt{2} f_{a_1} g_{a_1NN}}{m_{a_1}^2}, \tag{19}$$

with $g_A/g_V = 1.2694 \pm 0028$ and $f_{a_1} = (0.19 \pm 0.03)$ GeV^2 [15]. This value is close to the estimation of $g_{a_1pp} = 6.13 \sim 7.09$ that was obtained from the nucleon-nucleon potential in Ref. [18].

Because we are using the constituent quark model relation in Eq. (17), we use the constituent quark mass in the form factor of Eq. (9) for consistency. In the present work, we use $m_q = 0.27$ GeV that is supported by the study on hadron spectroscopy within Dyson-Schwinger equation approach [19].[2]

Shown in Fig. 2(a) are the results for the neutron δ_{LT}^n, while those for the proton δ_{LT}^p are given in Fig. 2(b). Here, the dashed lines are the predictions of the heavy baryon χPT of Ref. [1], which evidently overestimates the experimental data for δ_{LT}^n. The contributions from the a_1 exchange in Eq. (16) is given by the shaded areas because of the uncertainty of the coupling constants. As can be seen in Fig. 2(a), the a_1-exchange contribution to δ_{LT}^n is large and negative, so, when combined with the χPT calculation, it can bring down the theoretical prediction for δ_{LT}^n closer to the measured data of the Jefferson Lab E94010 Collaboration [3] than the χPT calculation. By the filled area in Fig. 2(a), we give the result that is obtained by combining the χPT prediction of Ref. [1] and the present work. This evidently shows the nontrivial role of the axial anomaly in δ_{LT}.

In the case of the proton, the a_1-exchange contribution to δ_{LT}^p has the opposite sign compared to the neutron case due to the isovector nature of the a_1-meson as can be seen in Fig. 2(b). As a consequence, the contribution of the a_1-exchange is added on the χPT prediction and the final result becomes larger. This is shown explicitly

[2] As the quark mass in the triangle graph becomes smaller, the contribution from the a_1 trajectory increases. For example, if we use $m_q = 170$ MeV as given by the mean field approximation in the instanton liquid model of Shuryak [20], it leads to the enhancement factor of $1.6 \sim 1.9$ to the form factor in the interval of $Q^2 = 0.1 - 0.26$ GeV^2. Such small value of constituent quark mass is supported, for example, by the NLO calculation of the hadronic contribution to muon anomalous magnetic moment [21]. In principle, one may try to use the running quark mass as a function of the quark virtuality k^2 in the triangle diagram as $m(k^2) = m_{\text{current}} + m_q(k^2)$. But this causes an additional unknown form factor in the a_1-quark vertex.

by the filled area in Fig. 2(b). Thus, measuring δ^p_{LT}, which is planned at the TJNAF [4], is an ideal tool to test the role of the axial anomaly to δ_{LT}.

However, different approaches of χPT give different predictions for δ_{LT} at low Q^2. As can be seen in Ref. [22], the predicted δ^n_{LT} of Ref. [2], which is based on a Lorentz-invariant formulation of χPT, has a very different Q^2 dependence. Namely, the predicted δ^n_{LT} and δ^p_{LT} of Ref. [2] decrease with Q^2 at low Q^2 region and then increase when $Q^2 \geq 0.05 \sim 0.1$ GeV^2, while those of Ref. [1] monotonically decrease with Q^2. This shows that δ_{LT} is very sensitive to the approach to hadron reactions. In Fig. 3, we give the results as in Fig. 2 but with the calculation of Ref. [2]. The results for δ^n_{LT} are shown in Fig. 3(a), while those for δ^p_{LT} are in Fig. 3(b). Since the contribution from the Δ resonance is hard to control, the predictions of Lorentz-invariant χPT are given by the shaded areas with dashed lines in Fig. 3 following Ref. [2]. This evidently shows that, although the two approaches of χPT give very different predictions on δ_{LT}, both of them overestimate the measured δ_{LT} of the neutron. Combining the a_1-exchange with the prediction of Ref. [2] leads again to a better agreement with the measured data for δ^n_{LT}. Thus, as can be seen in Figs. 2(a) and 3(a), the a_1-exchange has a crucial role to bring the χPT calculations of Refs. [1, 2] to the measured data for δ^n_{LT}, in particular, at low Q^2 region. However, we still overestimate the data at $Q^2 = 0.26$ GeV^2 for the both cases, and more elaborated examinations on the $Q^2 > 0.2$ GeV^2 region as well as on the χPT approaches are awaited.

Finally, we make a comment on the contribution of the flavor singlet $f_1(1285)$ axial-vector meson exchange to δ_{LT}. Because of the small coupling constant $g_{f_1NN} \sim 2.5$ [17] and the constituent quark model relation $g_{f_1qq} = g_{f_1NN}/3$, the contribution of the f_1-exchange is found to be an order of magnitude smaller than that of the a_1-exchange. This allows us to safely ignore the f_1-exchange contribution in the considered kinematical region.

In summary, we calculated the contribution of the axial anomaly to the generalized longitudinal-transverse polarizability δ_{LT} through the exchange of the a_1 Regge trajectory. In spite of the large uncertainties in the a_1 trajectory exchange, we found that its contribution is nontrivial and large enough to counterbalance the discrepancy between the χPT predictions and the experimental data for the neutron, especially, at low Q^2 region. To further test the role of the axial anomaly in the generalized longitudinal-transverse polarizability, we also presented the prediction for δ_{LT} of the proton, which will be measured at TJNAF in near future.

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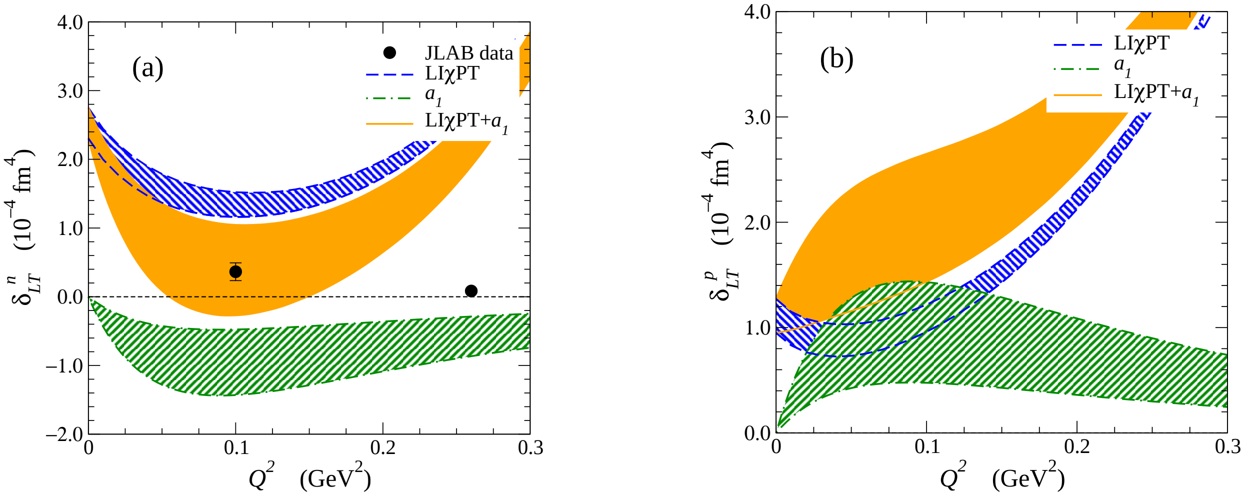

FIG. 3. (a) The generalized longitudinal-transverse polarizability of the neutron δ_{LT}^{n} and (b) of the proton δ_{LT}^{p}. Same as Fig. 2 but with the result of Lorentz-invariant χPT of Ref. [2].